A Pragmatic Justification of Many-Valued Logic

Omobola Olufunto Badejo
Department of Philosophy, Obafemi Awolowo University, Ile-Ife

Abstract
This paper explored a pragmatic justification of systems of many-valued logic. The paper traced the origin of many-valued logic to Aristotle’s challenge to the principle of bivalence based on the idea that the principle cannot accommodate contingent statements. The paper then examined the history of many-valued logic from Lukasiewicz, who rejected classical logic. The study evaluated some arguments on the possibility of systems of many-valued logic in Philosophy of Logic. The study employed the methods of logic. These included the analysis of the relevant concepts and an examination of arguments in philosophy of logic on the possibility of many-valued logic. Relevant literature on logic was consulted. This paper opines that the principle of bivalence had been misunderstood by some of the most influential proponents of many-valued logic, for example, Lukasiewicz. It was established that the terms true and false, in the arguments against bivalence, were used in an epistemic sense and not a logical sense. It was established that contrary to Aristotle’s and Lukasiewicz’s assumption, contingent found statements were necessarily either true or false; hence, the principle of bivalence could accommodate contingent statements. It was also established that many-valued logic did not warrant rejection of the principle of bivalence, or of classical logic, which was grounded in that principle. Consequently, both the principle of bivalence and many-valued logic could coexist in logic. The paper concludes that the most persuasive justification of many-valued logic is its pragmatic value and there was no conflict between it and the principle of bivalence in classical logic.

Key Words: Principle of Bivalence, Classical Logic, Many-Valued Logic, Contingent Statements.

Introduction
The traditional form and more widely accepted form of logic is classical logic. However, using Aristotle’s views on the nature of contingent statements and his rejection of the principle of bivalence, Jan Lukasiewicz developed an alternative to classical logic which he called a system of three-valued logic.1 Lukasiewicz is the first philosopher recorded to have created an alternative to classical logic and this is the origin of what is now known as many-valued logic.

Some scholars have rejected systems of many-valued logic on the grounds that Aristotle’s rejection of the principle of bivalence and Lukasiewicz’s adoption of it is erroneous. For them, the only acceptable form of logic is classical logic. They argue that the principle of bivalence is right and there is absolutely no need for another system of logic. I will argue in this paper that the principle of bivalence is right and should not be rejected, yet there is still a need for many-valued logic, not as an alternative to classical logic but as its necessary extension. The paper will argue that although many-valued logic started off, in some cases, on a wrong premise, it is a necessary and legitimate system of logic. The paper will further argue that contrary to the proponents of the first school of thought, classical logic and many-valued logic are compatible. This paper will conclude that there is a pragmatic justification for systems of many-valued logic.

Classical Logic and the Principle of Bivalence

Classical logic is a form of formal logic and probably the most popular and most studied form of logic. Classical logic is mainly in two forms: two-valued sentential calculus and predicate calculus. There are some principles and laws that characterise classical logic. Some of these principles are essential to classical logic. The fundamental or underlying principles of classical logic are the three basic laws of thought credited to Aristotle, and the principle of bivalence. These three laws of thought are: the laws of excluded middle, non-contradiction and identity.

The principle of bivalence states that for any statement P, P is necessarily either true or false. For example, it is necessarily true or false that “It is raining now”. Hence, classical logic allows for two possible truth-values: ‘true’ and ‘false’. If, however, it is proven beyond reasonable doubt that the principle of bivalence is false, then it follows that classical logic is equally false. There is hardly any disagreement on what the principle of bivalence asserts, but there are controversies on its truth. One of the earliest and most popular challenge to the principle of bivalence is Aristotle's essay on the problem of future contingents.

According to the proponents of classical logic, and Aristotle himself, the principle of bivalence is necessarily true and immutable; that is, it is binding and cannot be altered. However, Aristotle wrote an essay that created a doubt about the truth of the principle of bivalence. Aristotle never mentioned “bivalence” in his works. However, we can infer that

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3 Haack, *Philosophy of Logic*, 4.

4 These laws date back to the pre-Socratic period. However, Aristotle is regarded as the founder of the laws, in western philosophy. See Radim Belohlavek and George Klir;“From Classical Logic to Fuzzy Logic” In *Concepts and fuzzy Logic* (Massachusetts: MIT Press, 2011), 1-2.

he clearly had an understanding of this principle from his works. For Aristotle, it is a given fact that any declarative sentence should be necessarily either true or false. Aristotle also assumed that it is given that for any pair of contradictories; a statement and its negation, to be necessarily either true or false.

However, when we apply the principle of bivalence on statements that refer to future events, we run into the problem of future contingents. If all statements are necessarily either true or false, then the statement “There will be a sea battle tomorrow” must be necessarily either true or false. A pair of contradictories is such that when one of the pair is true, the other must be false. That is, when one of this pair of contradictories “There will be a sea battle tomorrow” and “There will be no sea battle tomorrow” is true then the other is false.

If the truth-value of a statement referring to a future event is determined as either true or false, prior to the actual occurrence of that event, the implication is that the event has been predetermined. We run into the problem of fatalism. Fatalism assumes that events are predetermined and their occurrences cannot be changed; that is, what will happen, will happen no matter what. If we can tell if a contingent statement is true or false, it follows that we can determine future events prior to their occurrence; then, fatalism is true.

However, Aristotle maintained that events are not predetermined. To avoid the problem of fatalism, we have to assume that contingent statements cannot be necessarily either true or false. To determine the truth-value of a contingent statement we must wait for the realisation or failure of realisation of the event that contingent statement refers to before we can determine its truth-value. If Aristotle’s argument is true it follows that principle of bivalence is not true after all. If there are some statements that are not necessarily either true or false, then it cannot be true that all statements are necessarily either true or false.

If the principle of bivalence states that every statement is necessarily either true or false, and the statement “There will be a sea battle tomorrow” cannot be known to be true or false till after the occurrence or non-occurrence of the sea battle, the principle of bivalence fails in respect to some statements. Aristotle highlighted the problem, but did not proffer any solution to it. From Aristotle’s essay on future contingents, we cannot infer any intention of laying a foundation for many-valued logics; he only intended to preserve the concept of freewill.

Aristotle thought that to say that the truth-value of statements referring to future events is restricted to truth and falsity, and that this therefore, is to pretend that the future is already determined and this would have unacceptable consequences on human life, especially with regards to ethics. However, if it is true that the principle of bivalence does not apply to statements about future events, how then can we determine the truth-value of

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7 John Ackrill and Lindsay Judson eds. *Aristotle Categories and De Interpretatione*, 55.
8 John Ackrill and Lindsay Judson eds. *Aristotle Categories and De Interpretatione*, 63.
such statements? Can contingent statements have truth-values at all? One of the attempts to answer these questions led to the creation of what we now know as many-valued logic.

**Lukasiewicz’s Three-Valued Logic**

Aristotle’s views on the nature of contingent statements provoked Jan Lukasiewicz into creating alternatives to logic of bivalence. Three-valued logic is the alternative to classical logic developed by Lukasiewicz.9 Some other proposed alternatives to classical logic are modal logic, temporal logic, fuzzy logic and default logic.10 Lukasiewicz, popular in the field of logic for his three-valued propositional calculus, is the first recorded logician to create a system of many-valued logic. With an adapted argument from Aristotle’s essay on the problem of future contingents, Lukasiewicz rejected the principle of bivalence and attempted to break away from classical logic. According to Lukasiewicz, if the principle of bivalence is true and applied to future tenses, we will reach a fatalistic conclusion. Lukasiewicz assumed that fatalism is unacceptable, and to prevent a commitment to fatalism he adopted a form of logic where the truth-value is more than the two traditional values - true and false.11

The language of Lukasiewicz’s three-valued logic is largely the same with that of logic of bivalence. However, they differ largely on the principle of bivalence; Lukasiewicz’s three-valued logic rejects the principle of bivalence. He introduced an additional truth-value: intermediate or possible. There are three truth-values: true, false and possible or intermediate. Lukasiewicz adopted Tarski’s definition of the third truth-value, “‘P is possible’ is defined ‘if non-P, then P’.”12 The third-value ‘possible’ is to suggest a more appropriate representation or the indeterminacy of the future. Lukasiewicz denoted the third-value; possible with ½, and maintained the traditional values for true and false; 1 and 0 respectively.

Lukasiewicz assumed that introducing a third value, possible, will address the problem of future contingents. If statements about future and past tenses are necessarily either true or false, the values of statements about future tenses are indeterminate or possible. The by-products of Lukasiewicz’s three-valued propositional calculus include the widely used truth-table in logic, mathematics and computer science, and the ‘polish notation’.13 The application of Lukasiewicz’s contribution to logic is influential in computer

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9 Bergmann, Lukasiewicz’s Three-Valued Logic, 76-79.
10 Default logic is proposed by Reiter Raymond to formalise default assumptions. There are some things we accept as true by default, despite the fact that they have exceptions, for instance, all birds fly. Default logic allows us to accept that all birds fly is true without stating the exceptions. Modal logic is a system of logic that formalises modalities such as ‘possibility’, ‘probability’ and ‘necessity’. Temporal logic is a system of logic that handles statements whose meaning or truth-value is quantified in time. For example, “I am hungry” is a statement that is meaning is constant in time and its truth-value can vary in time.
11 Priest, An Introduction to Non-Classical Logic, 243-244.
13 The polish notation is a symbolic logic invented by Jan Lukasiewicz in the 1920’s. The notation allows the order of operations and operands determine the result, this makes parentheses unnecessary. The
science, especially at the beginning of the computer era. Although, Lukasiewicz’s system recorded some successes, the theoretical foundation of the system is faulty. Aristotle and Lukasiewicz erred in assuming that the principle of bivalence is wrong.

Reactions to Aristotle and Jan Lukasiewicz

One objection to the foundation of Lukasiewicz’s three-valued system of logic is that just like Aristotle he did not make any attempt to argue that fatalism is highly undesirable before creating a system to avoid commitment to it. Both Aristotle and Lukasiewicz failed to argue for why fatalism is unacceptable and free will is acceptable. Aristotle and Lukasiewicz seem to accept that it is self-evident that fatalism is unacceptable and free will acceptable. Even if to determine the truth-value of a statement of a future event prior to its occurrence will indeed lead to fatalism, there is a need to argue that fatalism itself is undesirable and should be avoided. Aristotle and Lukasiewicz just assumed that there is a general understanding and agreement that fatalism is undesirable.

A second observation is that Aristotle and Lukasiewicz seems to misunderstand the principle of bivalence. The principle does not assert that all statements can be known to be true or false, but that all statements are necessarily true or false. Aristotle and Lukasiewicz confused an epistemic problem for a logical problem. That we cannot know whether a statement about future events is true or false is epistemic. That is, it shows one of the limits of human knowledge.

According to Quine, there is a difference between knowing that a statement is either true or false and knowing that a statement is true or a statement is false. Quine argued that any attempt to reject the principle of bivalence because of contingent statements will lead to a red herring. That is, the main point at issue will be neglected for another. The main issue is whether a contingent statement is necessarily either true or false, and not if it is necessarily true or necessarily false. The principle of bivalence is not disputing the possibility of having statements which we cannot establish to be true or to be false. The principle of bivalence states that statements are either true or false.

The creation of many-valued logic, Quine argued, stems from confusion over the difference between knowing that a statement is either true or false and knowing that a statement is true or that a statement is false. The logical interest in the truth-value of statements differs from the epistemological interest. The logician says “All statements are either true or false”. An epistemologist attempts to establish whether a statement is true or that it is false. Inability to establish that a statement is true or that it is false does not undermine the strength of the principle of bivalence. In other words, if I am unable to determine if an assertion is necessarily true or false, it has nothing to do with the ability of the statement to be necessarily either true or false. Aristotle and Lukasiewicz misunderstood removal of unnecessary parentheses and symbols simplifies the expression of logical and arithmetic relation.

14 Quine, *Philosophy of Logic*, 85

15 Ibid, 83
an epistemic problem for a logical problem. Consequently, contingent statements are necessarily true or false regardless of our epistemic limitations. It is, therefore, erroneous to reject the principle of bivalence on the basis of an epistemic limitation.

Furthermore, if I say that “It is true that there will be a sea battle tomorrow” today and tomorrow there is actually a sea battle, it does not follow that the event has been predetermined. There is nothing that shows that the event has been predetermined because I said it will happen and it actually happened. I could have guessed right anyway. What we know or do not know about the truth or otherwise of a statement has no bearing on its logical content. C.J.F. Williams has argued that contingent statements do obey the principle of bivalence. Some scholars, such as Aristotle and Lukasiewicz only assume they do not because they have a misconceived interpretation of the notion of truth.

According to Williams, it is a misconception to assume that truth is a real property that propositions can acquire or lose. For Williams, past tense statements about truth or falsity are not about the past, unlike other past tensed statements.\(^{16}\) A fatalistic conclusion cannot be inferred when a truth-value is ascribed to a declarative statement. For example, if I assert that “There will be a sea battle tomorrow”, and there is indeed a sea battle tomorrow, it does not follow that the truth-value of that statement has always been true. That is, “truth” in logic is a relation between words and not a relation between time and facts.\(^{17}\)

Williams argued that to infer from past truth to future necessity is a misconception of the word “truth” in logic. Hence, a contingent statement is not necessarily true just because prior to the occurrence of the event in the proposition someone had given its truth-value as true. Furthermore, it is not necessarily false just because someone had given its truth-value as false, prior to the occurrence of the event in that statement.\(^{18}\) The truth-value of a statement in logic is independent of persons.

Consequently, some philosophers such as C.J.F. Williams, James Tomberlin and Quine have rejected many-valued logic on the grounds that the premise on which the creation of the systems of many-valued logic lies, is wrong. For example, James Tomberlin argued that the assumption that Aristotle’s sea battle paradox will lead to fatalism is a fallacy, and there is no need to create a system to accommodate a fallacious assumption.\(^{19}\) Tomberlin highlighted Aristotle’s argument thus;

1. If/ it is necessarily true that there will be a sea battle tomorrow/, then/there will be a sea battle tomorrow/
2. There will be a sea battle tomorrow.
3. Therefore, necessarily, there will be a sea battle tomorrow.

\(^{16}\)Christopher Williams, “True Tomorrow, Never True Today.” \textit{The Philosophical Quarterly} 28, no. 113 (1978): 286.

\(^{17}\)See Dale Jacquette ed. \textit{Tarski’s Theory of Truth}, 86-102, for a discussion on the notion of truth in logic.

\(^{18}\)Williams, “True Tomorrow, Never True Today”, 295.

Let ‘it is necessarily true that there will be a sea battle tomorrow’ be \( \sim P \) and ‘there will be a sea battle tomorrow’ be \( P \), the argument can be rewritten as:

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\begin{align*}
(\sim P \rightarrow P) \\
P. : \sim P
\end{align*}
\]

The argument is clearly invalid and it is a formal fallacy to infer \( \sim P \) from the two premises\(^{20}\). The rule of modus ponens allows us to infer the consequent of a conditional statement when its antecedent is affirmed. To infer \( \sim P \) from \( (\sim P \rightarrow P) \) and \( P \) is a misuse of the rule of modus ponens and it is a fallacy of affirming the consequent.\(^{21}\) If the inference is a fallacy, it means that drawing a fatalistic conclusion from Aristotle’s argument is a fallacy. Hence, there is no need, at all, to try to create a system that can show that some statements, for instance, contingent statements are not necessarily either true or false, to avoid falling into the problems of pre-determinism and fatalism.\(^{22}\) For him, though systems of many-valued logics are interesting, there is absolutely no need to create them, since there is no problem their creation will address.\(^{23}\) If the fatalistic conclusion cannot be justifiably inferred from Aristotle’s argument, without committing a fallacy, there is no need to delve into the realm of any system of many-valued logic.

**Pragmatic Justification of Many-Valued Logic**

However, let us accept for the purpose of argument that it is true that deducing a fatalistic conclusion from Aristotle’s arguments, as suggested, is true. Would the problem of what the truth value of a contingent statement is have been solved? Rather than just discard many-valued logic, I will like to examine if there is any value in many-valued logic that can make it worth retaining. Is there some purpose that many-valued logic can serve that classical logic cannot serve? If we can get any, then it may be that the system is after all an advantage and not a problem in itself.

I argued earlier that there is nothing about contingent statements or statements that have vague objects and concepts within them to suggest that they are not capable of being either true or false. I argued that the challenges we face in determining whether the truth-value of these statements is more of an epistemic challenge or logical challenge. However, when we need to use these statements in formal reasoning, how do we handle them, despite the epistemic problems?

We have to examine three things; first, we have to determine whether there is any need for these statements; second, we have to see whether classical logic can handle them adequately, and third, if classical logic cannot handle them we have to see whether many-valued logic can handle them better. If many-valued logic can handle them adequately or

\(^{20}\)This fallacy is a misuse of the rule of modus ponens that states that from a conditional statement and the affirmation of its antecedent following the consequent. The formal fallacy highlighted by James Tomberlin is the fallacy of affirming the consequent.


\(^{22}\)Tomberlin, “The Sea Battle Tomorrow and Fatalism”, 353.

\(^{23}\)Ibid, 354.
better than classical logic, then there might be a pragmatic reason for studying many-valued logic. Rather than focus on the error of Aristotle and Łukasiewicz, philosophers can use some consequences of many-valued logic to show that logic is better off with systems of many-valued logic.

According to Haack, no system is completely closed and cannot be amended.24 Haack defended this argument by citing Quine who argued that:

No statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler suspended Ptolemy, or Einstein Newton, or Darwin Aristotle?25

Hence, if many-valued logic is rejected, it should not be because logic is a closed system that cannot be amended. It is because the system of many-valued logic itself is flawed or not justified and has no positive contribution whatsoever to formal reasoning.

Some proponents of many-valued logic assume that all systems of many-valued logic create additional values over the traditional ‘true’ and ‘false’. Haack argues that this is a wrong assumption. According to Haack, the most persuasive systems of many-valued logic do not actually create additional truth-values. These systems only create intermediary values or variants of ‘true’ and ‘false’ to address an epistemic problem. An example is Michalski’s twelve-valued logic.

Michalski’s twelve-valued logic system of logic is a computer programme that he designed to handle materials on some plant diseases. The twelve values do not reject bivalence. They are only used to classify truth-values economically. Something may be true of a plant disease in January, February, April and October and false of the same plant disease in other months. Rather than write all this information, Michalski created different variants of true and false to classify this information. What is true of a particular plant disease in four months can be established as the value ‘four’. Another information true of another plant disease in five different months can be established as the value ‘five’. These are not additional truth-values, but an economical way of saying that different things are true.26 Many-valued logic is not supposed to be a replacement for two-valued logic, but a measure used to address an epistemic problem, economise words used in a programme or to classify information. According to Haack, though some systems of many-valued logic attempt to challenge classical logic, it is largely as a result of misrepresentation of what the system of many-valued logic should address.27 The most persuasive and successful systems of logic, according to Haack, are those that recognise that the additional values are not actual values, but epistemic variants of ‘true’ and ‘false’.

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25Ibid, 25
26Haack, Philosophy of Logics, 214.
It has been argued that expressing information in declarative sentences is more modular than expressing it in segments of computer programs or tables. Furthermore, when we represent general facts about the world in logical statements, they can easily be used by any program.\textsuperscript{28} We are in the age where a computer is used to store and process facts about the world. Though computers are dispensable to a minority, in general terms, computers are indispensable. This is why it is important to provide a precise language for computers to understand. Since statements are a preferred means of intellectual activities involving reasoning and of representing information about the world, it is important that we handle them properly.

We encounter the challenge of determining if the truth-value of a declarative statement should be true or false when some information about that statement is not accurate, is absent, inconsistent or incorrect. When all information about a statement is accurate and present, then there can hardly be a challenge in determining whether the truth-value is true or false. If all we need to acquire knowledge and represent facts about the world are statements with complete information, then there is no need to study statements with incomplete information.

Organisations constantly require a wide variety of information, at one time or the other. They have to model some aspects of the real world. However, they tend to deal more with uncertain and inconsistent than certain information.\textsuperscript{29} For example, a financial trading company has to predict probable future performance of investments. It cannot survive just by dealing with the past and present performance of investments; it has to project into the future. Hence, it usually deals with contingent statements. An organisation deals with a lot of human concepts which are sometimes vague. The concepts and projections that organisation make use of might later become certain information, but at the time it is needed the information required to state the statements may be largely uncertain.

A hundred per cent consistency and certainty in human reasoning is almost a fairy tale. The world itself thrives largely on uncertainties and inconsistencies. Some people thrive largely on statements with uncertain information. For instance, a large number of people live on gambling. Some invest on betting, which has to deal mainly with probabilistic information. The use of statements with uncertain and inconsistent information is very wide. Engineering designs, auditing firms, production planning, law firms and market strategies, all require handling and making use of statements that do not have certain information at one time or the other. Thus, we cannot discard statements with uncertain and inconsistent information.

Consider this example, a new movie is about to be shown in a one-thousand five hundred sitting capacity cinema house. However, it is normal and expected of the


organisers of the show to sell more than one-thousand five hundred tickets for that particular show, despite the fact that it is inconsistent with the number of seats in the cinema house. At least no one expects two people to share a seat in a cinema house or carry each other, yet the organisers of the show sell more tickets than the available number of seats. This is because it is likely that they make more profit from the show if handled that way. They can assume that some clients will buy the tickets without showing up for the show.

Uncertainties and inconsistencies may sometimes lead to difficult situations. However, they are important to business and intellectual activities. When we consider statements with uncertain or inconsistent information, we have to deal with some statements that we cannot immediately classify as either true or false. Some may come out both true and false, some may be neither of the two at the time the statement is being used. However, classical logic reasons with statements that are necessarily either true or false. Hence, it may be necessary to look beyond classical logic to formalise those statements. The ideal way of handling statements with incomplete information is to either use some assumptions or add some information to them.

Many-valued logic extends the notion of truth and falsity to accommodate these perspectives in some statements. One major value of classical logic lies in its certainty. Proponents of many-valued logic do not deny that classical logic has been precise in the use of language and certain in its application. What differentiates classical logic from any other alternative to it is its assumption that statements are either true or false. I have argued earlier on that even when we find it difficult to immediately determine whether a statement is either true or false, it does not follow that it is not capable of being either true or false. In other words, there is no contradiction in the assumption of classical logic that statements are either true or false, even though some statements cannot be immediately classified as either true or false.

Disjunctive information is used in classical logic to handle statements with incomplete or uncertain information. In some statements where there are two alternatives and we are not sure of which is the right alternative, classical logic can use the disjunction to add an information to the proposition or represent the statement. Hence, there will be more information added to the statement and the statement then becomes more useful in knowledge acquisition or data collection and collation.

For instance, suppose that I need to collate a data where the age of Barak Obama should be represented and the only information I have is that he is in his early forties. The information I have is incomplete and I need some more information to make it useful. I can use disjunctive information to represent his age for the purpose of the collation of that information. That is, I can say “Obama is 40 v 41 v 42 v 43 v 44 v 45 v 46 v 47 v 48 v 49 years old”, which is a true statement since Barack Obama is over 40. This use of the disjunctive information strengthens the information in the statement and makes the data collation easier.

However, when the information required strengthening a statement cannot be covered with disjunctive information, classical logic will not be able to handle the
statement. Classical logic cannot handle probabilistic information, default information and fuzzy information. Default information is general laws, not universal laws. Defaults are not hundred per cent (100%) accurate, but they can be used to make useful inferences where the available information is inadequate. Fuzzy information is the use of vague concepts in reasoning, when the available information is inadequate or adhering to precision is difficult. For example, to precisely establish who falls into a category of tall men is difficult.

Another form of statements that cannot be immediately classified as true or false consists of statements with likelihoods, because they are largely intuitive and probabilistic. However, likelihood is a very powerful tool for reasoning in several firms, especially firms that deal with market research. Probabilistic information is usually acquired through experiments and research, but the information must be refined to make them workable. Probabilistic information is very important, but must be well handled. In any organisation where likelihood or estimated information is needed, the use of probabilistic information is necessary. For example, if a company needs to introduce a new product into the market, it needs information on how the public will react to it. However, it has to project and present plans that can show that the product is viable on the market.

Statements with probabilistic information require more than disjunctive information to make them workable. The best disjunctive information can do is to say that the success of a new product is viable or the success of the product is not viable. However, for a company trying to present plans and strategies in order for its new design or product to be accepted, disjunctive information is almost useless. To make such statements more relevant, there is need for information that will suggest that the prospect of the new product being successful is very high.

Another set of statements with uncertain or incomplete information are statements with vague concepts. For example, it is easier to specify a woman than to specify who a beautiful woman is. To decide whether a woman is beautiful or not may lead to controversies, since there are different standards, both objective and subjective, for making decisions based upon vague concepts. A concept is said to be vague when it is capable of imprecision when used or when its meaning or use is unclear.

It seems that though human beings struggle to arrive at precision at different times, we thrive a lot on imprecision. For example, it is usually easier for a tailor to design an outfit when there are no specific instructions by the customer on what outfit to make. The tailor simply does what he or she assumes will fit the customer, but is easy to make. However, when the customer gives precise instructions, difficulties in meeting the instructions precisely increase. In other words, at different times the success of our daily activities thrive on vagueness and imprecision.

When we have to deal with statements such as “There is a woman approaching now”, classical logic can handle the truth-value easily. However, when we have to deal with statements such as “There is a beautiful woman approaching now”, we cannot say with

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30 Hunter, 65-75.
certainty the truth-value of the statement. Statements with vague concepts, though they are capable of being true or false, are usually not handled by classical logic. However, statements with vague concepts can be handled by fuzzy logic. These are instances where many-valued logic can address epistemic limitations in reasoning.

Conclusion

Proponents of many-valued logic are correct on the interpretation they gave to Aristotle’s essay in the chapter nine of his De’Interpretatione. Aristotle actually challenged the truth of the principle of bivalence. However, some of the proponents of classical logic are also right to argue that Aristotle and some of these proponents of many-valued logic failed to understand that although there are difficulties in determining whether some declarative statements are true or false, this does not imply that such statements are not capable of being true or false.

The argument that the principle of bivalence is limited or false, as argued by Lukasiewicz and Aristotle, is not right. The principle of bivalence is right to assert that all statements are necessarily either true or false. But it is wrong to assume that the principle of bivalence also means that we can know, at all times and immediately, when a statement is true or false. The principle of bivalence is the logical core of classical logic and should not be jettisoned. Classical logic is a logic in its own right. There is nothing wrong with creating systems that can handle statements that we cannot establish as true or false. But this is not equivalent to a rejection of the principle of bivalence.

The best argument that can be used to justify systems of many-valued logic is its pragmatic value. The pragmatic value of many-valued logic is a sufficient condition for its existence. The systems of many-valued logic should stand not as a challenge to the principle of bivalence, but as various attempts to address the difficulties faced in applying bivalence to some declarative statements. When we face difficulties in determining whether some statements are true or false, it becomes almost impossible to do any form of formal reasoning with that statement. Many-valued logic then helps make these kinds of statements amenable to formal reasoning, rather than abandon the statements.

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